July 2021

Cabot Learning Federation

I am a mathematician…

I am a mathematician. I am curious and wish to solve problems but I know the answer is only the beginning. I recognise that getting stuck is part of problem-solving and that it helps me develop the resilience and strategies to persevere.

In order to become fluent in recognising, representing and communicating about mathematical concepts, I actively explore them through talk with others; making conjectures, justifying my ideas and generalising from specific examples to create an increasingly efficient and connected understanding. I apply this understanding and mathematical habits of mind to find patterns that help me break into, make sense of and break down novel problems to find solutions.

I reflect on what I have done to improve my strategies, collaborating with others to construct a shared understanding that increasingly helps us to make sense of the world, giving us enjoyment, agency and a sense of self and place.

"Mathematics could be like roller-skating, but usually it's like being told to stop roller-skating and come in and tidy your room."

Richard Winter (1991)

What is mathematics?

Mathematics is the science that deals with the logic of shape, quantity and arrangement. It can be seen as the search for meaningful pattern and structure that can be applied to model and solve problems in every facet of human enquiry. Its substantive knowledge has evolved from simple counting, measurement and calculation but its disciplinary knowledge (including skills of inquiry, conjecture and proof) forms an integral part of the study and application of mathematics.

Curriculum expectations

In the English national curriculum, outcomes in mathematics go beyond recalling facts and emulating procedures, as emphasised by the three aims of the curriculum and in the expectation that:

> *"By the end of each key stage, pupils are expected to **know**, **apply** and **understand** the matters, skills and processes specified in the relevant programme of study."*
>
> (DfE, 2014)

Our curriculum intends to make explicit the content knowledge and the necessary processes and skills involved in learning mathematics so that teachers and leaders help all learners achieve the three aims of the national curriculum and to demonstrate all of the elements of "*mathematical proficiency*" (National Research Council, 2001) – *see image, right*.

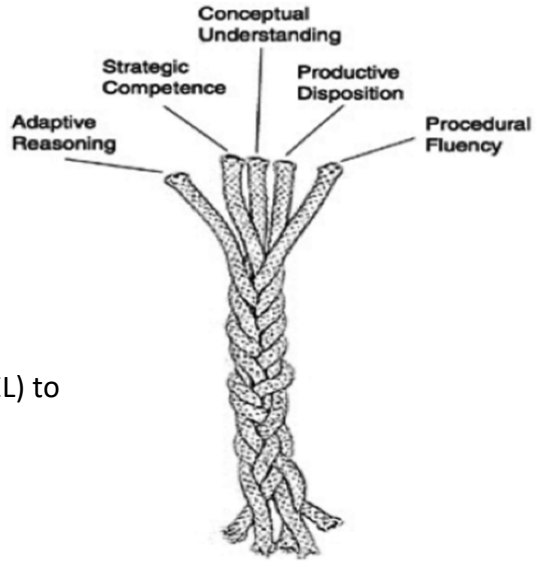
Learners learn and apply mathematical processes as they work to achieve the expectations outlined in the curriculum. They actively engage in applying these processes, together with social-emotional learning skills, across the curriculum to support learning in mathematics.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- connecting
- communicating
- representing
- selecting tools and strategies

They are the means through which all learners acquire and apply mathematical knowledge, concepts, and skills. In the Cabot Learning Federation we are developing a **habits of mind** approach to develop the mathematical processes and social-emotional learning skills for mathematics. This builds on the Early Years' Characteristics of Effective Learning (CoEL) to develop all learners as proficient mathematicians ready to engage in and enjoy mathematics.

The image on the following page is intended to demonstrate this offer for learners; how the content of the curriculum is accessed, developed and applied through social-emotional learning skills and the mathematical processes.



Strands of mathematical proficiency

Mathematics curriculum expectations for all learners

Social-emotional learning skills

- Mathematical resilience
- Collaboration
- Developing a sense of self and sense of place
- Thinking critically & creatively

Mathematical Habits of Mind

Mathematical processes

- Problem solving
- Reasoning & proving
- Reflecting
- Connecting
- Communicating
- Representing
- Selecting tools & strategies

See 'Curriculum rationale' in Appendix D

Number	Ratio & proportion	Algebra	Geometry & Measures	Statistics
• Number sense • Place value • Calculations • Fractions	• Relations between quantities	• Patterns • Early algebraic thinking • Variables & expressions	• Lines, shapes & solids • Position & direction • Measurement	• Data collection & organisation • Data visualisation • Data analysis

Mathematical content – "*matters, skills and processes*"

Mathematical Habits of Mind

The basis of our habits of mind approach is a set of "innate learning powers" available to all learners, through which, over time, everyone is able to make sense of their environment to understand complex concepts and learn to perform skills and processes. Key examples of 'independent' learning posited by educators (including Caleb Gattegno and John Mason) demonstrating the existence of these powers include the *development of language* and *learning to walk* by young children.

Our intent is to make these innate powers explicit and harness them as efficient strategies for learning mathematics and solving problems by establishing them as deliberate thinking habits. In addition to these natural ways of thinking, further strategies of organising (being systematic), reflecting and extending problems are intended to lead to improved problem solving and deeper understanding of content. These strategies and useful attitudes encompass the social-emotional learning skills and mathematical processes. However, as attitudes to mathematics – both in the UK generally and in many of our learners' specific contexts – can often be less than positive, we have chosen to have a particular emphasis on mathematical resilience.

See Appendix A for further guidance on our habits of mind approach. In 2020/21 and 2021/22 the focus has been / will be on developing teaching and learning for reasoning, which encompasses the three innate learning powers of **conjecturing, convincing** and **generalising**. Future development work will focus incrementally on remaining aspects.

Mathematical resilience

Mathematical resilience acknowledges social and emotional responses to the subject that need to be anticipated and managed to support enjoyment and success. There are four key aspects in building mathematical resilience:

- having a growth mindset; believing that mathematical capability can improve through effort with effective strategies;
- knowing that maths can have personal value, is of value in the world and that the learner is valued as a mathematician (e.g. in their class);
- knowing that learning maths can involve struggle and 'getting stuck' and that this is to be expected rather than avoided;
- knowing how to find and enlist support to stay in the 'growth zone'

This article provides a useful model for developing mathematical resilience in the classroom.

Mathematical content

The mathematical content of the curriculum reflects the Early Years Foundation Stage and is broadly aligned with the national curriculum programmes of study (PoS). However, the DfE/NCETM non-statutory guidance (Ready to Progress criteria) indicates a sensible prioritisation of the primary content, following two years of disrupted schooling. Based on sound pedagogical reasons, some of the prioritised content is suggested to be introduced later than in the national curriculum PoS, which presents a dilemma for schools, while national assessments align to the content of the PoS. In 2021/22, we will prioritise teaching of the Ready to Progress (RtP) criteria with individual academies making decisions about additional content based on their learners' needs.

The ***suggested*** yearly overviews (long term plans) for single year group cohorts for Years 1-6 can be found in Appendix C - Part 1; they provide guidance on sequencing and time allocation with links to the relevant Big Ideas (see below) and RtP criteria.

CLF Primary Mathematics Big Ideas

> "Close examination of lesson planning and teachers' thoughts about lesson planning in education systems where pupils do well reveal an intense focus on underlying knowledge structures and connections rather than the surface coherence of activities and teaching. This means that teachers are planning for what pupils will be thinking about or with, not what they will be 'doing'."
>
> OFSTED Research Review Series: Mathematics (2021)

To support teachers and learners in making connections to core mathematical structures and seeing mathematics as a connected whole, rather than a series of disparate parts, we have identified **Big Ideas** (key concepts) in mathematics, defining and describing them for teachers. Over time, these will be exemplified from Nursery to Year 6, linked directly to the appropriate content from Development Matters and Birth to 5 Matters (for Early Years) and the RtP criteria (for Primary) to support coherence in teaching of these curricular priorities. *Age-appropriate interpretation and detailed exemplification for Nursery to Year 6 in Appendix C – Part 2*.

Number: cardinality (including subitising and counting) – understanding that each numeral represents a quantity. Counting (using one-to-one correspondence) is one way of *enumerating* – establishing how many things are in a set – because the last number you say when you have counted them all correctly is the *cardinal value* of the set. However, we have innate ability to subitise (recognise how many without counting) small quantities and this is the basis for developing early understanding of number through linking quantity, numeral and number name.

Number: composition (including conceptual subitising) – understanding that one quantity can be made up from (composed from) two or more smaller quantities. Learning to 'see' a whole number and its parts at the same time is a key development in number understanding and starts with noticing smaller parts within wholes of up to five.

Number: comparison (including comparison of quantities and position in the linear number system) – knowing which numbers are worth more or less than each other. Starting with comparing sets of objects, as learners' understanding of cardinality develops, they are able to compare abstract numbers and to understand their relative positions in a linear number system. Comparison and equivalence extend to more abstract mathematical ideas in later years.

Number: estimation – building on the innate ability to perceive differences in quantity (Approximate Number System), learners develop an increasingly accurate approximate sense of magnitude.

Number: place value – Understanding that our number system uses ten digits (0-9) and a place value system that enables an infinite set of numbers to be represented as numerals. The numeral of a 1-digit number consists of one digit; the numeral of a 2-digit number consists of two digits etc. Place value is a

complex construct (with some aspects that are not developmentally appropriate content for KS1), including: positional aspect, additive aspect, base ten aspect and multiplicative aspect.

Number: the additive relationship – The additive relationship can be described as the relationship between two parts and their sum (the whole) - in other words it describes the relationships in any given composition (or decomposition) of a quantity. "*The sum of two numbers is a third number which contains as many units as the other two numbers taken together. When two of these three values are known, the third can be found*." Ma & Kessel (2018)
Number: the multiplicative relationship – The multiplicative relationship describes three quantities connected by a one to many correspondence or ratio; when two are known the third can be found. "*The problem children face is that they view multiplication as a unary operation where you only have to really think about one number – the number being added or taken away. Seeing multiplication as repeated addition can exacerbate this unary view. Multiplication must be seen as a binary operation with two distinct inputs or elements in the process and a ratio maintained. Multiplication is framed by coordinating two ideas/quantities simultaneously.*" Ma & Kessel (2018)

Number: proportion – Continuous quantities that cannot be represented by a single whole number can be represented using decimal fractions or common fractions. Meaningful use of either format requires understanding of the quantities they represent. Fractions can also represent quantities involving a ratio between two other quantities (e.g. the concentration of orange juice in a jar can be described by the ratio of orange concentrate to water; the probability of an event can be described by the ratio between the number of favourable cases to the total number of cases)

Number: properties – for every set of numbers there are relationships that are always true and they give rise to rules that govern arithmetic and algebra. Developing age-appropriate knowledge of number properties is a key aspect of number sense.

Pattern – relationships can be described and generalisations made for mathematical situations that have numbers or objects that repeat in predictable ways. Seeking patterns and making conjectures based on them is the foundation of all true mathematical activity.

Lines, shapes & solids – two- and three-dimensional objects with or without curved lines and surfaces can be described, classified, and analysed by their attributes and this, rather than naming, should be the focus of teaching in early geometry.

Spatial thinking – objects in space can be oriented in an infinite number of ways and an object's location in space can be described quantitatively; objects in space can be transformed in an infinite number of ways and those transformations can be described and analysed mathematically

Data – some questions can be answered by collecting and analysing data and this data can be represented in different ways

Measurement – some attributes of objects are measurable and can be quantified using unit amounts.

The history of mathematics – the discipline of mathematics has a rich history that can help us to interpret and understand current conventions and help learners to connect to their own sense of self and sense of place as mathematicians.

Assessment

The main purpose of assessment activities is to identify next steps for learners to develop all aspects of their mathematical proficiency. Therefore, the majority of assessment must be formative and ongoing. The different expectations in the statutory frameworks (EYFS vs. national curriculum) lead to different approaches for assessment, although the whole child perspective from Early Years is one that we are now increasingly taking account of in primary.

Early Years

When assessing children's maths in the Early Years, it is important we take time to get to know them as learners and as mathematicians. What do they already know? What skills do they already have? How do they approach problems and what mathematical attributes do they possess? This is important in any age group, but especially with our youngest learners. Take time to observe their play; what spontaneous mathematical language are they using? Can they notice patterns? Do they have a sense of spatial awareness?

The characteristics of effective learning are fundamental to assessing young learners' mathematics. As well as the knowledge and skills we want children to develop, building mathematical resilience, an inquisitive mind and 'willingness to have a go' are key habits of mind for mathematicians. Most importantly, the focus needs to be on **what learning you see embedded in their play**, rather than what they can recall in a formal situation.

Appendix E – Part 1 contains further guidance and some indicative milestones for maths in Nursery and Reception, culminating in the (statutory) Early Learning Goals for the end of Reception.

Primary

As in Early Years, understanding what children already know and can do is key. Teaching incorporates assessment for learning, supported by day-to-day classroom activities (including peer- and self-assessment) that promote dialogue and encourage learners to share their mathematical thinking and reasoning, as indicated by the national curriculum:

> *"The national curriculum for mathematics reflects the importance of spoken language in pupils' development across the whole curriculum – cognitively, socially and linguistically. The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.* ***They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions.****"*
>
> (DfE, 2014)

In line with our curriculum expectations, we assess content knowledge through application with social-emotional learning skills and the mathematical processes. Day to day tasks and activities must require thought from learners and create challenge (even struggle) in order to provide holistic assessment information, whilst, additionally, supporting learning.

In addition, teachers might use diagnostic assessments prior to teaching (to check for prerequisite understanding) and use low-stakes summative assessments at various intervals to check for recall and accuracy of appropriate facts and skills. While these may inform aspects of teaching (and there is evidence that they can also support retention), they cannot inform holistic assessment of mathematicians, since this requires the use of rich tasks that engage social-emotional learning skills and mathematical processes. Use of timed, summative assessments should be balanced against our curriculum expectations that aims for all learners to achieve the three aims of the national curriculum and attain mathematical proficiency.

Summative DOOYA assessments will be supported through moderation meetings (facilitated in Terms 2 and 5) and shared Trust-wide tasks for learners (see below). They are intended to support teachers making judgements as to whether pupils are on track to meet the end of year expectations outlined in the transition criteria (Appendix E – Part 2). However, the majority of assessment information for most learners will come from their work in class, when they demonstrate their ability to reason about age-appropriate content and to apply age-appropriate content to solve problems.

Expectations for learners participating in national assessments (KS1 SATs, Y4 MTC and KS2 SATs) and for teachers providing teacher assessment judgements will continue to be set by the Standards and Testing Agency and aspects of our Trust-wide work will provide support for these.

Shared Trust-wide tasks

To supplement the reasoning and problem-solving activities teachers plan for in the daily maths lessons, each (short) term will see the Trust-wide provision of an age-appropriate task for each year group Y1-6. Video footage of learners engaging in these activities will provide exemplification for reasoning (and, over time, other habits of mind) and will form the basis for our twice-yearly moderation meetings. The activities are optional and teachers will make decisions about whether to use them whole-class or with focus groups (e.g. those that require additional assessment information). However, in order to develop our collective expertise in assessing reasoning, in 2021/22, teachers will need to bring footage from their classrooms to the moderation meetings in November and May.